Probabilistic AHP and TOPSIS for Multi-Attribute Decision-Making under Uncertainty

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Daniel Guggenheim School of Aerospace Engineering
Est. 1930

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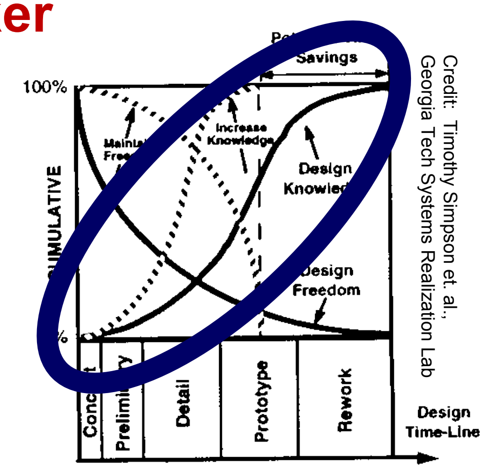
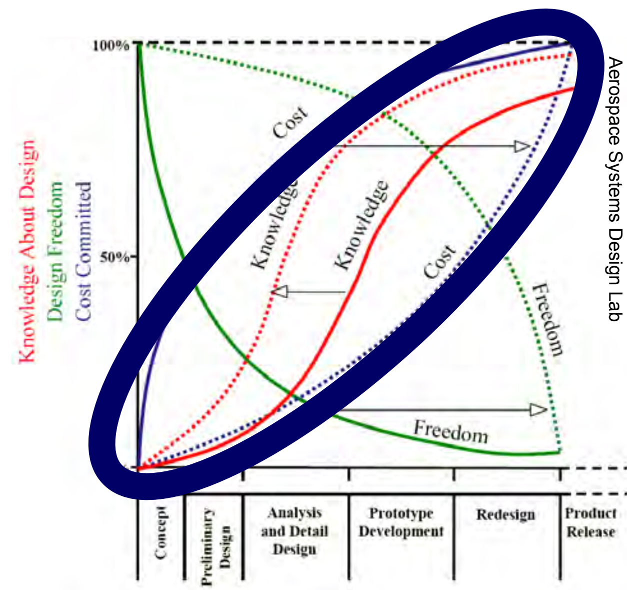
Introduction

Making informed design decisions in the face of uncertainty is a common challenge in aerospace systems engineering.

Uncertainty in Decision-Maker Preferences and System Performance



Credit: Timothy Simpson et. al., Georgia Tech Systems Realization Lab



Credit: Dimitri Mavris, Georgia Tech Aerospace Systems Design Lab

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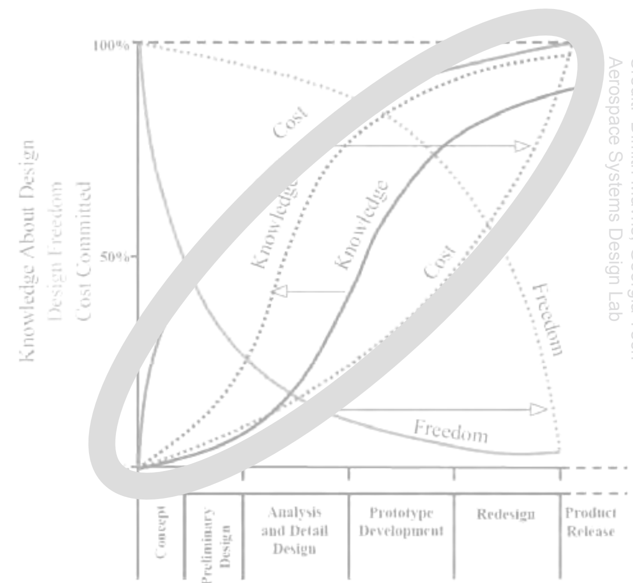
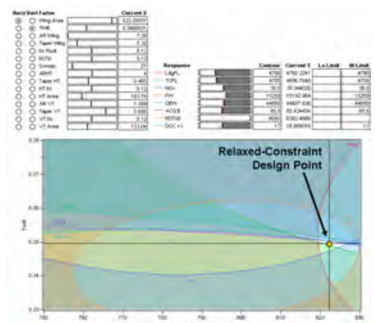
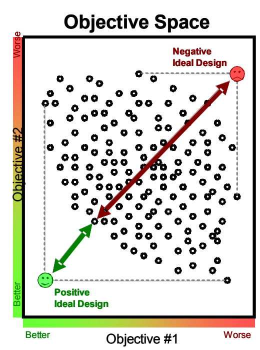


Trades among Multiple Objectives



	Objective #1	Objective #2	Objective #3	Objective #4	Objective #n
Objective #1	1	2	5	2	1/3
Objective #2	1/2	1	7	1	1
Objective #3	1/5	1/7	1	1/5	1/8
Objective #4	1/2	1	5	1	1
Objective #n	3	1	8	1	1

	De	De	De	Design #4	Design #k
Design #1	1	1	7	3	1/7
Design #2	1	1	3	2	3
Design #3	1/7	1/3	1	1/9	1/2
Design #4	1/3	1/2	9	1	5
Design #k	7	1/3	2	1/5	1



Introduction

Common Tools for Multi-Attribute Decision-Making

- Analytic Hierarchy Process (AHP)
- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

AHP

- Developed by Saaty in mid-1970s
- Permits both prioritization of objectives and selection of designs based on a ***structured approach of pairwise comparisons***
- Matrices are reciprocal and permit decision-maker(s) to evaluate consistency of preferences
- Using AHP for concept selection equivalent to simple additive weighting
- Conventional AHP is deterministic

	Objective #1	Objective #2	Objective #3	Objective #4	Objective #n
Objective #1	1	2	5	2	1/3
Objective #2	1/2	1	7	1	1
Objective #3	1/5	1/7	1	1/5	1/8
Objective #4	1/2	1	5	1	1
Objective #n	3	1	8	1	1

Priority Vector
0.251
0.208
0.036
0.193
0.312

Objective #n	Design #1	Design #2	Design #3	Design #4	Design #k
Design #1	1	1	7	3	1/7
Design #2	1	1	3	2	3
Design #3	1/7	1/3	1	1/9	1/2
Design #4	1/3	1/2	9	1	5
Design #k	7	1/3	2	1/5	1

Introduction

Common Tools for Multi-Attribute Decision-Making

- Analytic Hierarchy Process (AHP)
- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

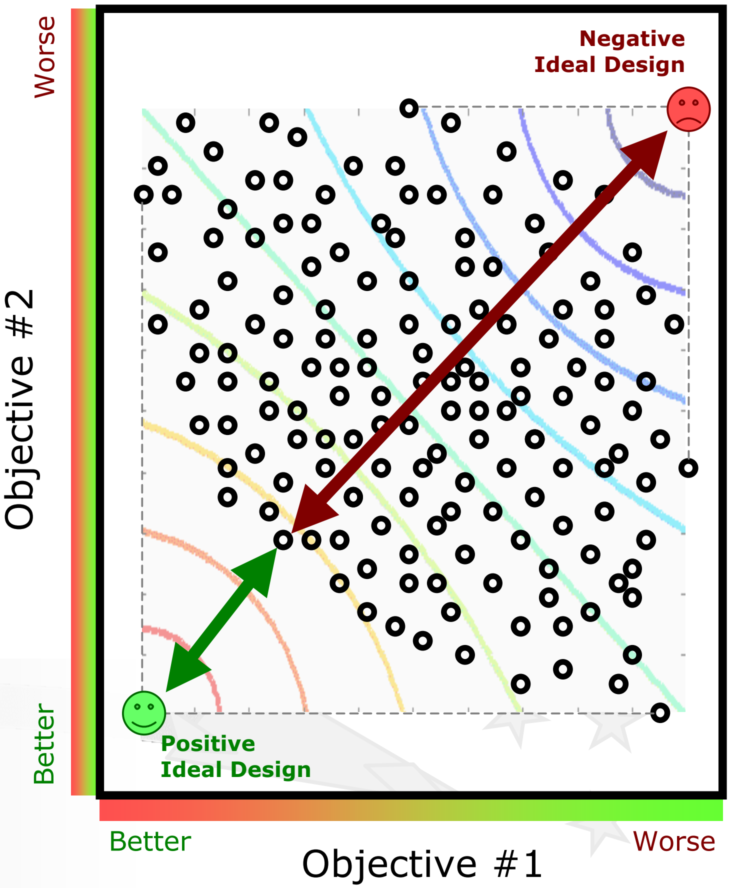
TOPSIS

- Developed by Yoon & Hwang, 1980
- Scores alternatives based on euclidean distances from positive and negative ideal designs

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-} = \frac{\left\|\vec{d}_i - \vec{d}_-\right\|}{\left\|\vec{d}_i - \vec{d}_+\right\| + \left\|\vec{d}_i - \vec{d}_-\right\|}$$

- Intuitive "physical" meaning
- Reflects diminishing marginal rates of substitution
- Does not provide method for weighting objectives (can be coupled with AHP)

TOPSIS Objective Space



Introduction

Objective

Facilitate informed decisions under uncertainty in decision-maker preferences by developing probabilistic adaptations of AHP and TOPSIS

Example Aerospace Application

Choose ***launch vehicle*** and circular ***orbit*** for a small (400 kg), responsive military reconnaissance satellite

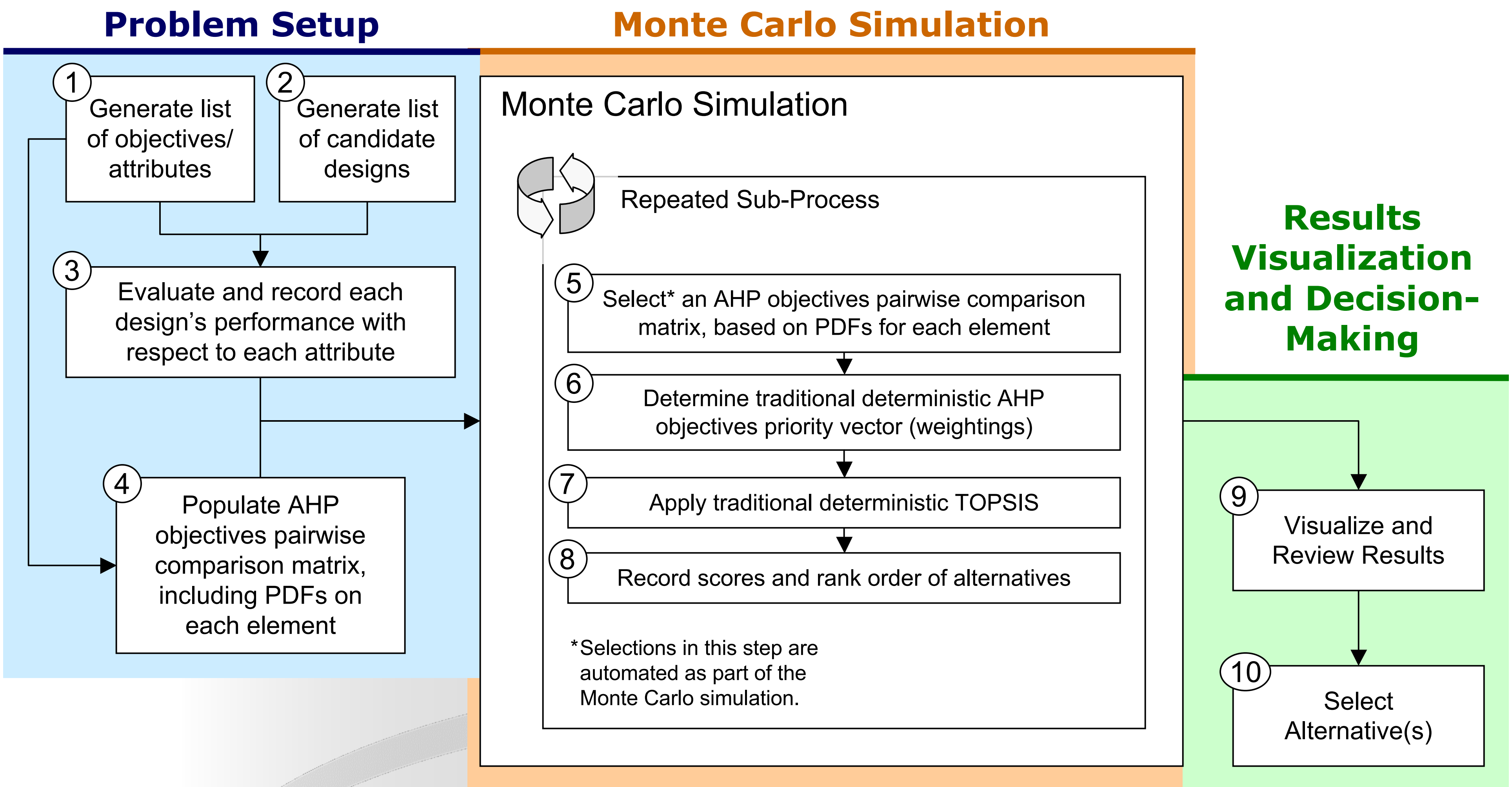
- Targeted sensor with 1° FOV angle and 1.0 m GSD at 400 km reference altitude
- 110 kg/m² ballistic coefficient, minimal propellant available for orbit maintenance

Modeled after **TacSat-2**
Launched Dec. 16, 2006

Credits: USAF (top,right)
NASA (left)

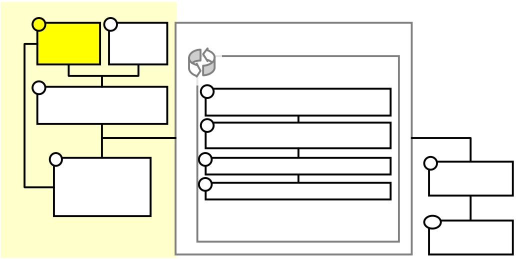
Methodology

Methodology Summary



Methodology: Problem Setup

Step 1: Generate List of Objectives/Attributes

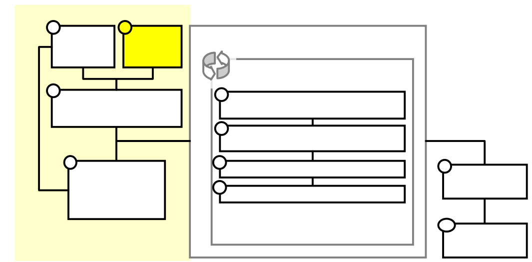


What does the decision-maker wish to achieve?

Attribute	Units	Preferred Value
Launch Margin	percent	High
Launch Cost	$FY2009M	Low
Launch Reliability	percent	High
Image Field of View Area	km^2	High
Image Nadir Ground Sample Distance	km	Low
Mean Worst-Case Daily Data Latency	hours	Low
Mean Daily Coverage Time	hours	High
Orbit Lifetime	years	High

Methodology: Problem Setup

Step 2: Generate List of Candidate Designs



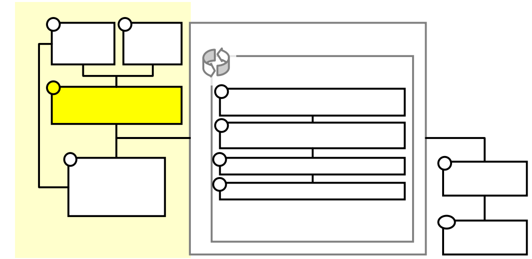
With what design choices can these objectives be influenced?

Design Variable	Options Considered
Launch Vehicle	Falcon 1, Falcon 1e, Pegasus XL, Pegasus XL with HAPS, Taurus 2110, Taurus 2210, Taurus 3110, Taurus 3210, Minotaur I, Minotaur IV, Athena I, Athena II
Orbit Altitude (km)	200, 300, 400, 600, 1000, 1500, 2000
Orbit Inclination (deg.)	0, 10, 20, 30, 40, 50, 60, 70, 80, 90

840 Discrete Design Options

Methodology: Problem Setup

Step 3: Evaluate each Design's Performance with respect to each Attribute



Attribute	Units	Evaluation Technique
Launch Margin	percent	ELV RSEs
Launch Cost	$FY2009 M	LV Database
Launch Reliability	Percent	LV Database
Image Field of View Area	km²	Geometry
Image Nadir Ground Sample Distance	km	Geometry
Mean Worst-Case Daily Data L[illegible]	[illegible]	[illegible]
Mean Daily Coverage Time	[illegible]	[illegible]
Orbit Lifetime	[illegible]	[illegible]

Filters:

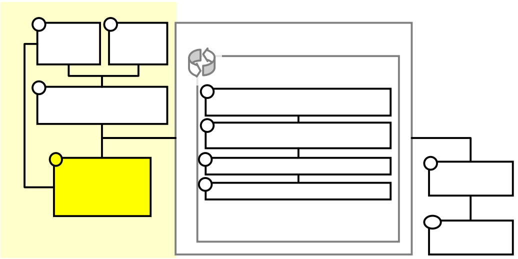
Launch Margin >0% and <100%
Mean Daily Coverage >0 hrs.
Orbit Lifetime >3 months

Design Definition				Design Attributes							
Design No.	Orbit Alt. (km)	Orbit Inclin. (deg.)	Launch Vehicle	Launch Margin (percent)	Launch Cost ($FY09M)	Launch Reliability (percent)	Image FOV Area (km²)	Image Nadir GSD (m)	Mean Worst-Case Daily Data Latency (hrs)	Mean Daily Coverage Time (hrs)	Orbit Lifetime (yrs)
1	400	30	Pegasus XL	0.1	22.4	93.6	38.3	1.0	14.8	0.62	0.5
2	400	30	Minotaur I	31.9	22.4	97.9	38.3	1.0	14.8	0.62	0.5
[illegible]	400	30	Athena I	86.4	42.4	95.2	38.3	1.0	14.8	0.62	0.5
[illegible]	400	40	Minotaur I	27.4	22.4	97.9	38.3	1.0	14.1	1.00	0.5
[illegible]	400	40	Athena I	76.9	42.4	95.2	38.3	1.0	14.1	1.00	0.5
[illegible]	400	50	Minotaur I	22.8	22.4	97.9	38.3	1.0	12.8	1.18	0.5
[illegible]	[illegible]	50	Athena I	67.1	42.4	[illegible]	38.3	1.0	12.8	1.18	0.5
8	[illegible]	60	Minotaur I	18.0	22.4	[illegible]	38.3	1.0	11.7	1.10	0.5
9	[illegible]	60	Athena I	56.9	42.4	[illegible]	38.3	[illegible]	11.7	1.10	0.5

59 Candidate Designs

Methodology: Problem Setup

Step 4: Populate AHP Prioritization Matrix, including Uncertainties



Baseline AHP Prioritization Matrix

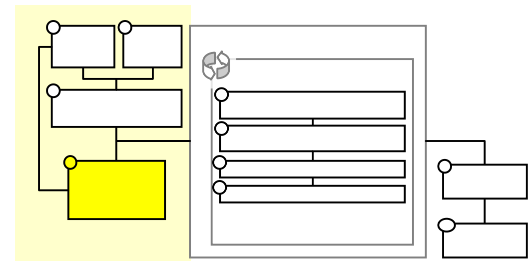
	High Launch Margin	Low Launch Cost	High Launch Reliability	High Image FOV Area	Low Image Nadir GSD	Low Mean Worst-Case Daily Data Latency	High Mean Daily Coverage Time	High Orbit Lifetime
High Launch Margin	1	1/3	3	5	1/6	1/3	1/2	3
Low Launch Cost	3	1	5	7	1/4	1/3	1/2	5
High Launch Reliability	1/3	1/5	1	3	1/8	1/5	1/4	1/5
High Image FOV Area	1/5	1/7	1/3	1	1/9	1/8	1/7	1/7
Low Image Nadir GSD	6	4	8	9	1	4	5	7
Low Mean Worst-Case Daily Data Latency	3	3	5	8	1/4	1	2	5
High Mean Daily Coverage Time	2	2	4	7	1/5	1/2	1	3
High Orbit Lifetime	1/3	1/5	5	7	1/7	1/5	1/3	1

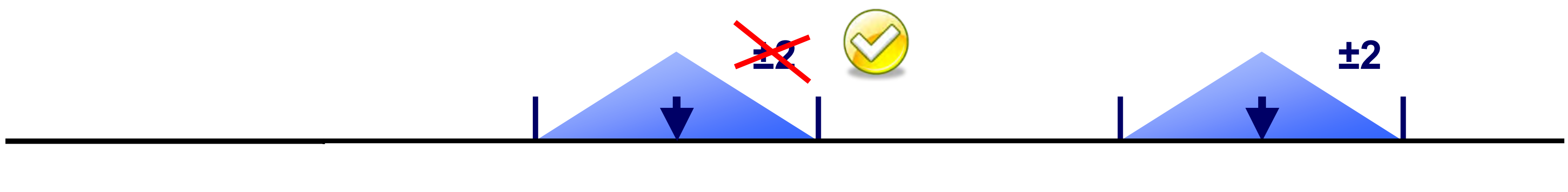
Verbal Description of Importance Ratings*

1 = Equal
3 = Weak
5 = Strong
7 = Very Strong
9 = Absolute

* *T.L. Saaty, The Analytic Hierarchy Process, New York: McGraw-Hill, 1980.*

Step 4: Populate AHP Prioritization Matrix, including Uncertainties





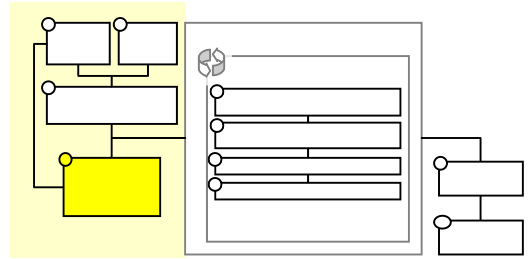
$$v(a)=\begin{cases}\dfrac{2a-1}{a} & \text{if } a<1\\ a & \text{if } a\geq 1\end{cases}$$

$$a(v)=\begin{cases}\dfrac{1}{2-v} & \text{if } v<1\\ v & \text{if } v\geq 1\end{cases}$$

Uncertainty Matrix (Virtual Scale)

	High Launch Margin	Low Launch Cost	High Launch Reliability	High Image FOV Area	Low Image Nadir GSD	Low Mean Worst-Case Daily Data Latency	High Mean Daily Coverage Time	High Orbit Lifetime
High Launch Margin	0	3	1.5	2	2	2.5	2.5	2.5
Low Launch Cost		0	1.5	2	2	3	3.5	4
High Launch Reliability			0	3	1	1.5	1.5	5.5
High Image FOV Area				0	0	0.5	0.5	1.5
Low Image Nadir GSD					0	2	2	1.5
Low Mean Worst-Case Daily Data Latency						0	0.5	0.5
High Mean Daily Coverage Time							0	1.5
High Orbit Lifetime								0

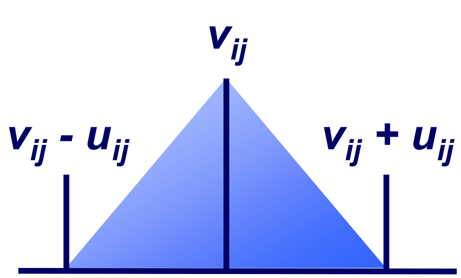
Methodology: Problem Setup



Step 4: Populate AHP Prioritization Matrix, including Uncertainties

Probabilistic Prioritization Matrix (Virtual Scale)

	High Launch Margin	Low Launch Cost	High Launch Reliability	High Image FOV Area	Low Image Nadir GSD	Low Mean Worst-Case Daily Data Latency	High Mean Daily Coverage Time	High Orbit L
High Launch Margin	1	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	$X_{1,6}$	$X_{1,7}$	X
Low Launch Cost	$2-X_{1,2}$	1	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	$X_{2,6}$	$X_{2,7}$	X
High Launch Reliability	$2-X_{1,3}$	$2-X_{2,3}$	1	$X_{3,4}$	$X_{3,5}$	$X_{3,6}$	$X_{3,7}$	X
High Image FOV Area	$2-X_{1,4}$	$2-X_{2,4}$	$2-X_{3,4}$	1	$X_{4,5}$	$X_{4,6}$	$X_{4,7}$	X
Low Image Nadir GSD	$2-X_{1,5}$	$2-X_{2,5}$	$2-X_{3,5}$	$2-X_{4,5}$	1	$X_{5,6}$	$X_{5,7}$	$X_{5,8}$
Low Mean Worst-Case Daily Data Latency	$2-X_{1,6}$	$2-X_{2,6}$	$2-X_{3,6}$	$2-X_{4,6}$	$2-X_{5,6}$	1	$X_{6,7}$	$X_{6,8}$
High Mean Daily Coverage Time	$2-X_{1,7}$	$2-X_{2,7}$	$2-X_{3,7}$	$2-X_{4,7}$	$2-X_{5,7}$	$2-X_{6,7}$	1	
High Orbit Lifetime	$2-X_{1,8}$	$2-X_{2,8}$	$2-X_{3,8}$	$2-X_{4,8}$	$2-X_{5,8}$	$2-X_{6,8}$	2-X	



$$X_{ij} \sim Triangular\left(v_{ij} - u_{ij}, v_{ij}, v_{ij} + u_{ij}\right)$$

$$X_{ji} = 2 - X_{ij}$$

Methodology: Monte Carlo Sim

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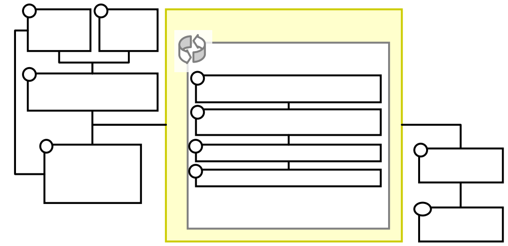

For each Simulation:

- **Step 5: Select an AHP Prioritization Matrix**
- **Step 6: Determine AHP Priority Vector**
- **Step 7: Compute TOPSIS Score**
- **Step 8: Record Scores, Ranks of Alternatives**

Dispersed Prioritization Matrix

	High Launch Margin	Low Launch Cost	High Launch Reliability	High Image FOV Area	Low Image Nadir GSD	Low Mean Worst-Case Daily Data Latency	High Mean Daily Coverage Time	High Orbit Lifetime
High Launch Margin	1	1/3	3	5	1/6	1/3	1/2	3
Low Launch Cost	3	1	5	7	1/4	1/3	1/2	5
High Launch Reliability	1/3	1/5	1	3	1/8	1/5	1/4	1/5
High Image FOV Area	1/5	1/7	1/3	1	1/9	1/8	1/7	1/7
Low Image Nadir GSD	6	4	8	9	1	4	5	7
Low Mean Worst-Case Daily Data Latency	3	3	5	8	1/4	1	2	5
High Mean Daily Coverage Time	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
High Orbit Lifetime	1/3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-} = \frac{\left\|\vec{d}_i - \vec{d}_-\right\|}{\left\|\vec{d}_i - \vec{d}_+\right\| + \left\|\vec{d}_i - \vec{d}_-\right\|}$$

Design Definition				Scores according to Baseline Prioritization	
Design No.	Orbit Alt. (km)	Orbit Inclin. (deg.)	Launch Vehicle	Closeness, C_i	Rank
1	400	30	Pegasus XL	0.5546	27
2	400	30	Minotaur I	0.5603	23
3	400	30	Athena I	0.5574	25
4	400	40	Minotaur I	0.5694	18
5	400	40	Athena I	0.5657	21
6	400	50	Minotaur I	0.5783	12
7	400	50	Athena I	0.5739	16
8	400	60	Minotaur I	0.5817	11
9	400	60	Athena I	0.5765	14
10	400	70	Falcon 1e	0.6166	3
11	400	70	Minotaur I	0.59[illegible]	7
12	400	70	Athena I	0.59[illegible]	10
13	4[illegible]	[illegible]	Falcon 1e	[illegible]	2
[illegible]	[illegible]	[illegible]	Mi[illegible]r I	[illegible]	[illegible]

Methodology: Decision-Making

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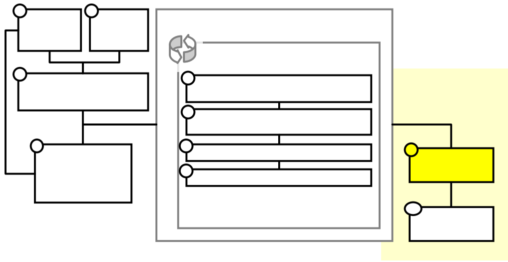
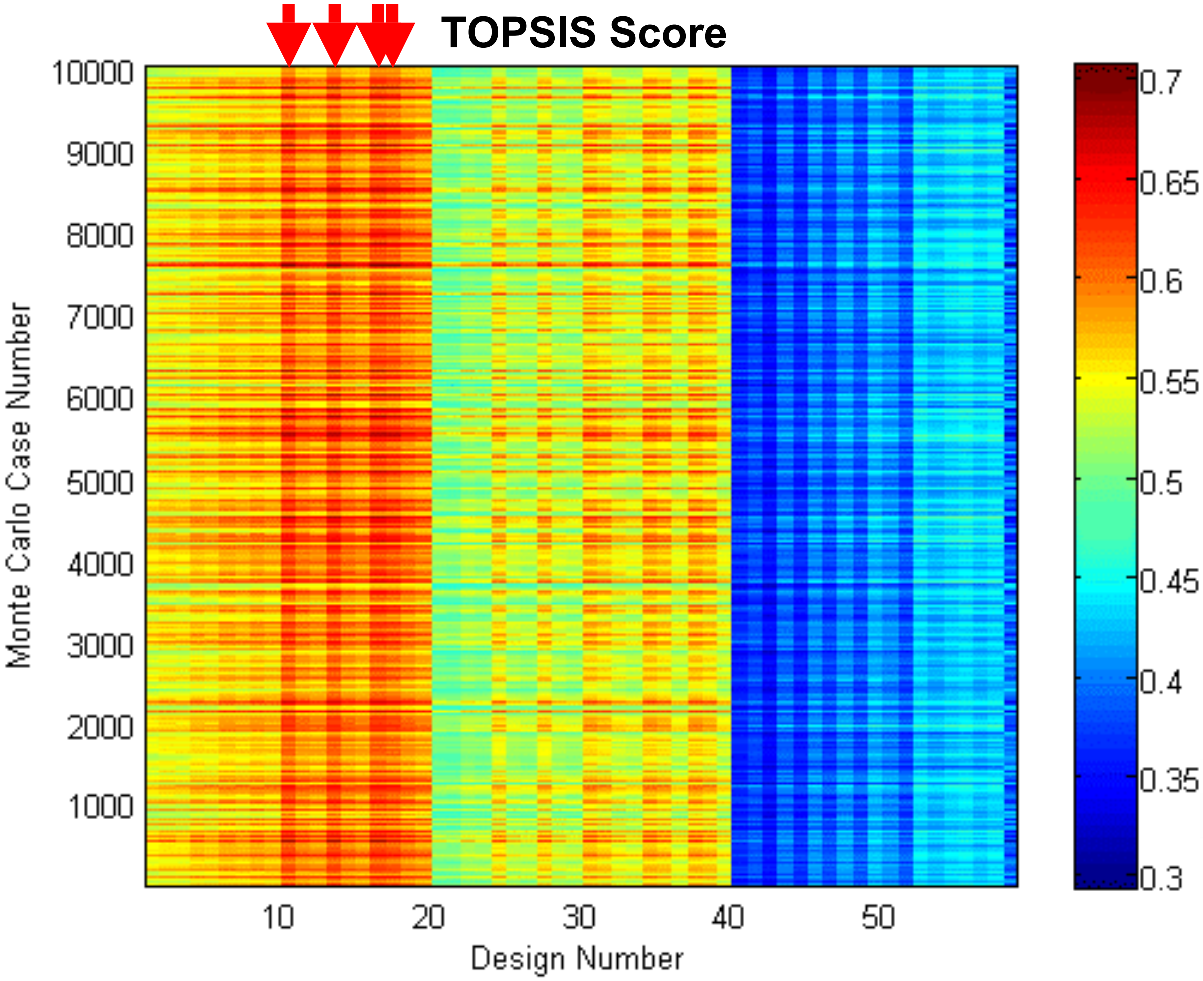
Step 9: Visualize and Review Results

1	2	3





Falcon 1e – 400 km, 70°

Falcon 1e – 400 km, 80°

Falcon 1e – 400 km, 90°

Taurus 2210 – 400 km, 90°

Methodology: Decision-Making

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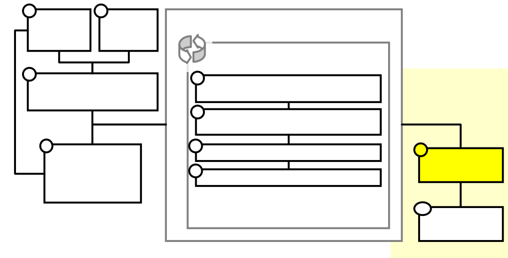
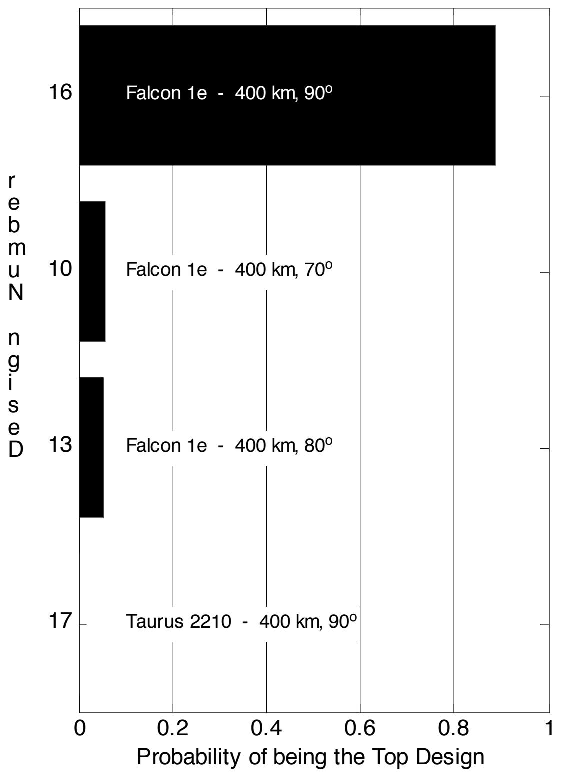
Step 9: Visualize and Review Results

1	**2**	3



Among the 10,000 Monte Carlo runs, how often does a particular design appear as the best alternative?

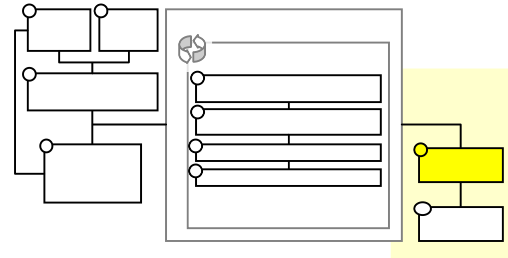


Probability	Deterministic TOPSIS Score
88.71%	**0.6194**
5.82%	**0.6166**
5.42%	**0.6183**
0.05%	**0.6134**

Methodology: Decision-Making

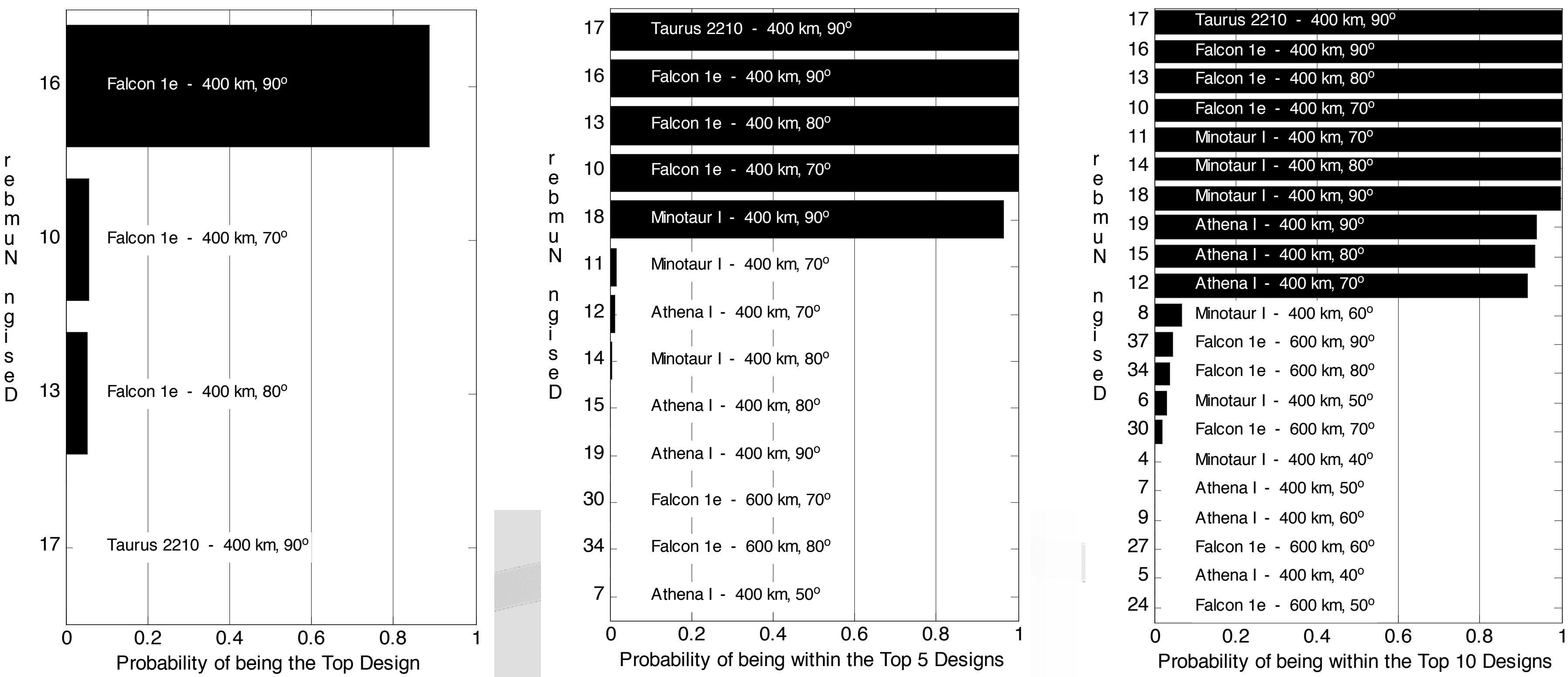
Step 9: Visualize and Review Results

1	**2**	3



Among the 10,000 Monte Carlo runs, how often does a particular design appear among the top N alternatives?

***N* = 1** ***N* = 5** ***N* = 10**



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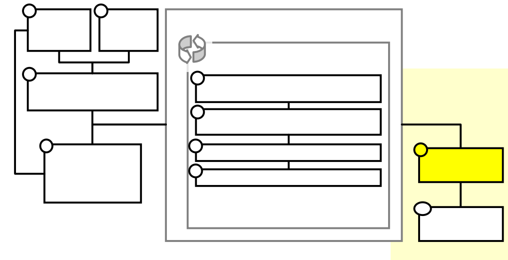
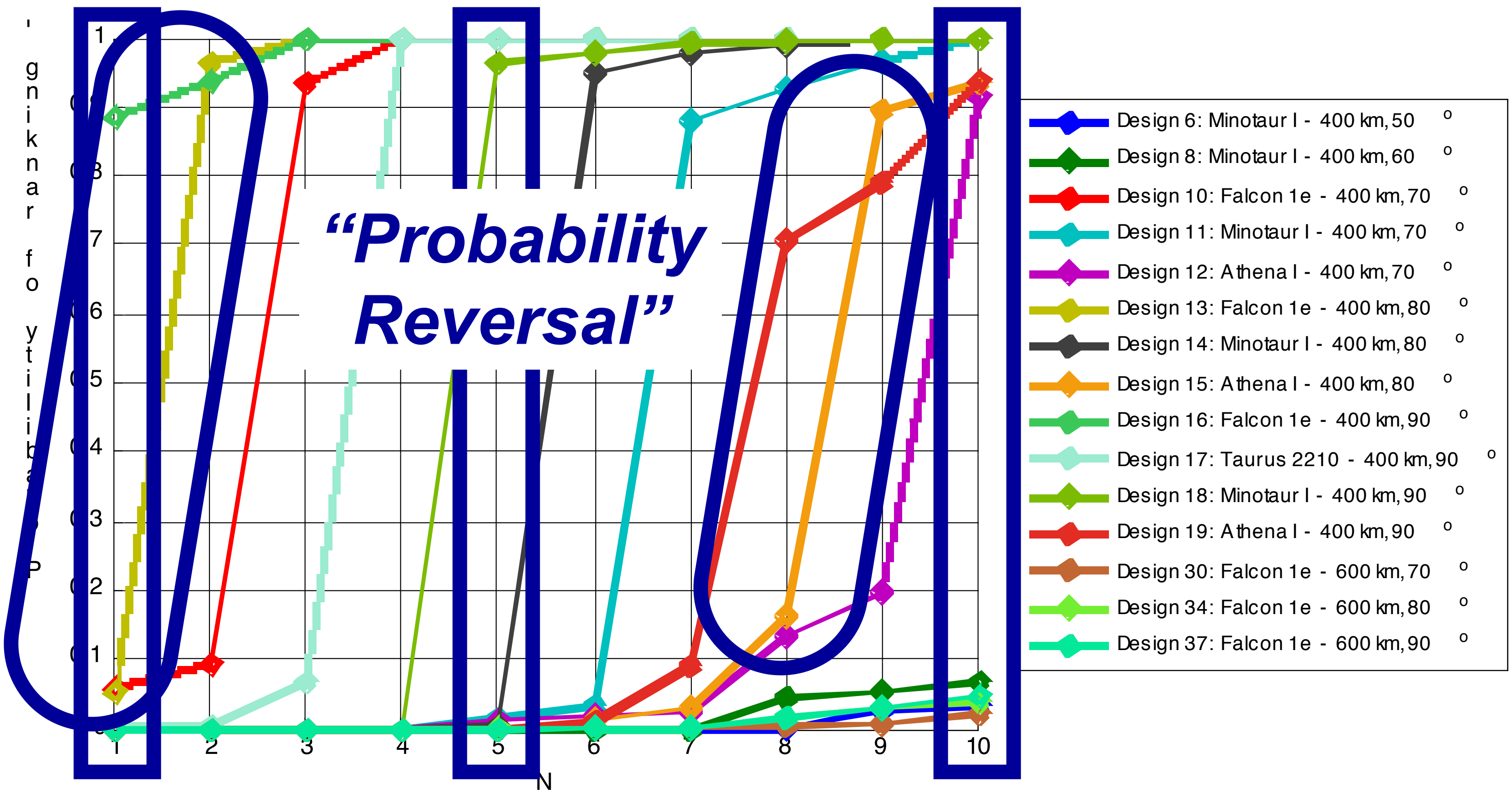
Step 9: Visualize and Review Results

1	2	3

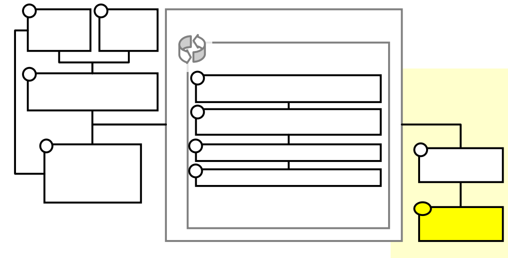


Among the 10,000 Monte Carlo runs, how often does a particular design appear among the top N alternatives?



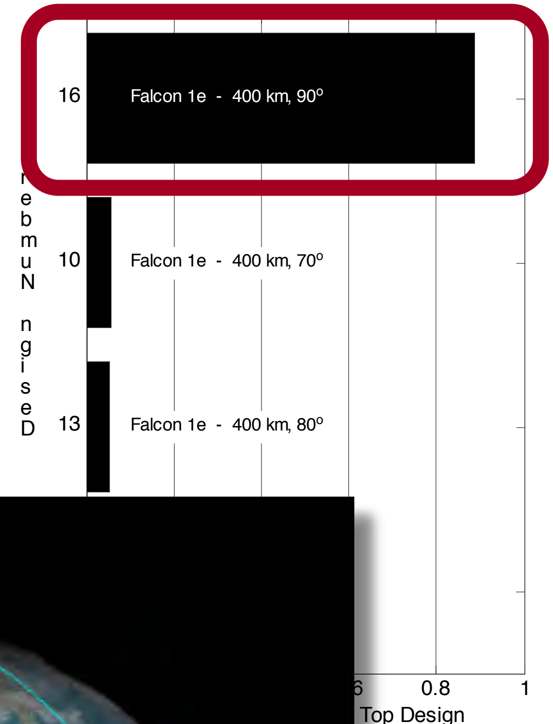
Methodology: Decision-Making

Step 10: Select Alternative(s)



Considerations

- Did particular design characteristics appear consistently in top designs?
- Is finer discretization on continuous variables necessary, and is there time for a second iteration?
- Is selection intended to result in a single design solution or a *family* of solutions?



Conclusion

- Probabilistic extension of AHP and TOPSIS can facilitate more informed engineering decisions under uncertainty
  - Adds easily interpretable results to characterize effects of uncertainty in preferences
  - May be particularly useful if multiple designs must be selected
  - Demonstrated on practical engineering application
- Comprehensive method consists of 10 steps and 3 segments:
  - **Problem Setup**: Definition of objectives, priorities, uncertainties, design attributes, and candidate designs; and evaluation of each design with respect to attributes
  - **Monte Carlo Simulation**: Application of traditional AHP and TOPSIS to thousands of "dispersed" AHP prioritization matrices
  - **Results Visualization and Decision-Making**: Examination of results through up to four techniques to inform final decision
- Areas for Expansion
  - Incorporation of design attribute uncertainty (model uncertainty)
  - Incorporation of nonsymmetric or non-triangular PDFs

Questions?

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For more information, visit
http://www.nasa-academy.org/soffen/travelgrant/

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