A two equation Eulerian—Lagrangian model for simulation of shock induced flow through particle suspension

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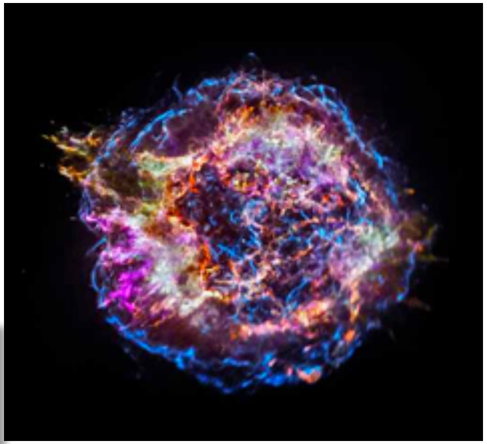
This work is funded by Early Stage Innovation (ESI) Fund from NASA

Applications of particle-laden compressible flows

Nature

volcanic eruptions

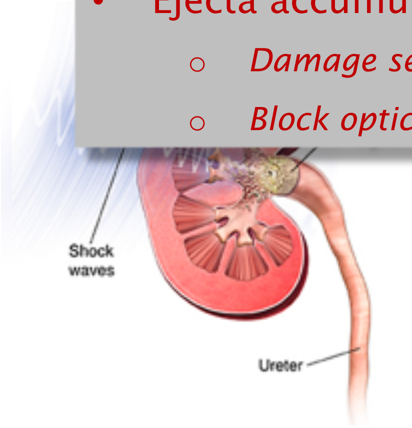
supernovas



Risks caused by plume-surface interaction:

- Surface fluidization and site alteration
- Asymmetric back pressure due to cratering
- Ejecta accumulation and erosion
  - *Damage sensors*
  - *Block optical access*

Engineering



shock wave lithotripsy

coal dust explosions

plume-surface interactions

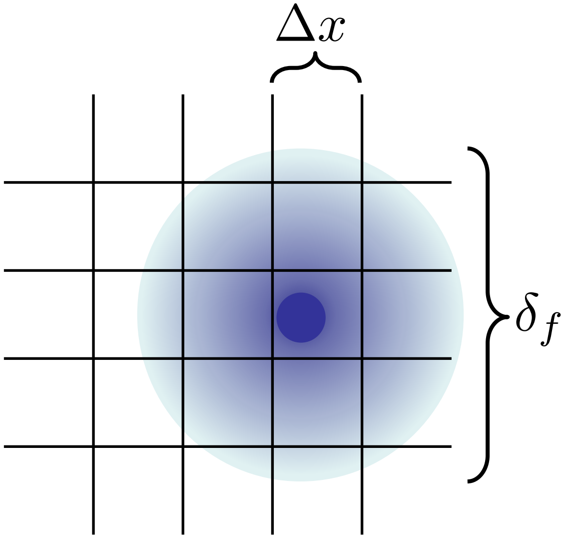
Volume-filtered description of two-phase flows

Objective: *formulate equations for fluid-particle flows that enable Δx>d_p*

- Introduce a local volume filter based on convolution product with kernel:

$$\overline{A}(\boldsymbol{x}) = \int_V A(\boldsymbol{y}) G(|\boldsymbol{x}-\boldsymbol{y}|)\,\mathrm{d}\boldsymbol{y}$$

- Accurate solution requires $\Delta x \ll \delta_f$



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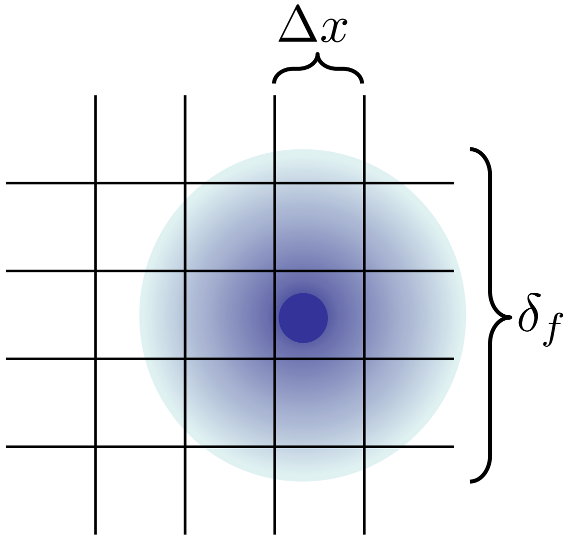

Mass: $$\frac{\partial\alpha\bar{\rho}}{\partial t} + \nabla\cdot(\alpha\bar{\rho}\widetilde{\boldsymbol{u}}) = 0$$

$$\boldsymbol{\mathcal{F}} = \sum_{i=1}^{N_p}\int_{\mathcal{S}_i} \boldsymbol{n}\cdot(p'\mathbb{I}-\boldsymbol{\tau}')\,\mathcal{G}(\|\boldsymbol{x}-\boldsymbol{y}\|)\,d\boldsymbol{y}$$

Momentum exchange (drag, lift, etc.)

Momentum: $$\frac{\partial\alpha\bar{\rho}\widetilde{\boldsymbol{u}}}{\partial t} + \nabla\cdot(\alpha\{\bar{\rho}\widetilde{\boldsymbol{u}}\otimes\widetilde{\boldsymbol{u}}+\boldsymbol{R}_u\}) = \alpha\nabla\cdot(\overline{\boldsymbol{\tau}}-\bar{p}\mathbb{I}) + \boldsymbol{\mathcal{F}}$$

$$\boldsymbol{R}_u = \bar{\rho}(\widetilde{\boldsymbol{u}\otimes\boldsymbol{u}} - \widetilde{\boldsymbol{u}}\otimes\widetilde{\boldsymbol{u}})$$

‘Pseudo-turbulent’ Reynolds stress

Volume-filtered description of two-phase flows

Energy

$$\frac{\partial \alpha \overline{\rho} \widetilde{E}}{\partial t} + \nabla \cdot \left(\alpha \overline{\rho} \widetilde{E} \widetilde{\boldsymbol{u}} \right) + \nabla \cdot \left(\alpha (\overline{p} \widetilde{\boldsymbol{u}} - \widetilde{\boldsymbol{u}} \cdot \overline{\boldsymbol{\tau}}) \right) + \alpha \nabla \cdot \overline{\boldsymbol{q}}$$

$$= - \left(\overline{p} \mathbb{I} - \overline{\boldsymbol{\tau}} \right) : \nabla \left(\alpha_p \overline{\boldsymbol{u}}_p \right) + \overline{\boldsymbol{u}_p \cdot \boldsymbol{\mathcal{F}}} + \mathcal{Q} - \nabla \cdot \left(\alpha \left\{ \cancel{\boldsymbol{R}_{Eu}} + \boldsymbol{R}_{uu} - \cancel{\boldsymbol{R}_{u}} \right\} \right)$$

$$\boldsymbol{R}_{uu} \approx \boldsymbol{u} \cdot \boldsymbol{R}_u$$

$$\mathcal{Q} = \sum_{i=1}^{N_p} \int_{\mathcal{S}_i} \boldsymbol{n} \cdot \boldsymbol{q}' \mathcal{G}(\|\boldsymbol{x} - \boldsymbol{y}\|) \; d\boldsymbol{y}$$

Heat exchange (Nusselt number correlation)

Thermodynamic relation:

$$\widetilde{E} = \frac{\overline{p}}{\overline{\rho}(\gamma - 1)} + \frac{1}{2} \widetilde{\boldsymbol{u}} \cdot \widetilde{\boldsymbol{u}} + \frac{1}{2} tr \left(\boldsymbol{R}_u \right)$$

Pseudo-TKE systematically acts to reduce pressure (and local sound speed), and thus increase local Mach number

Volume-filtered description of two-phase flows

Energy

$$\frac{\partial \alpha\bar{\rho}\widetilde{E}}{\partial t} \ldots$$

$$= -(p\mathbb{I} - \tau) : \nabla(\alpha_p \boldsymbol{u}_p) + \boldsymbol{u}_p \cdot \mathcal{F} + \mathcal{Q} - \nabla\cdot(\alpha\{\cancel{\boldsymbol{R}}_u + \boldsymbol{R}_{uu} - \cancel{\boldsymbol{R}}\ldots\})$$

- *PTKE can be 30-100% of resolved KE, and is f(Ma_s, Re_p, α_p)!*
  - *Important contribution in modeling.*

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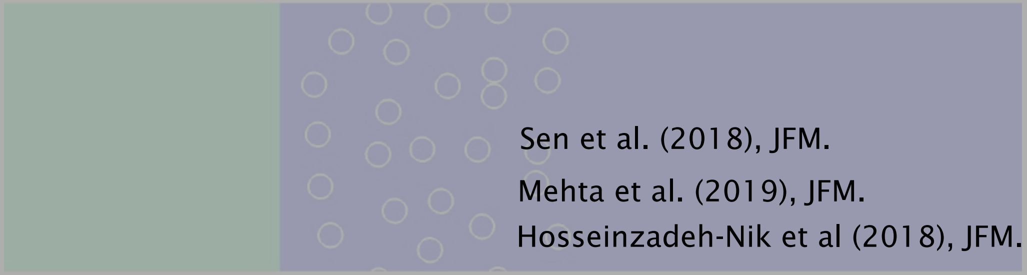


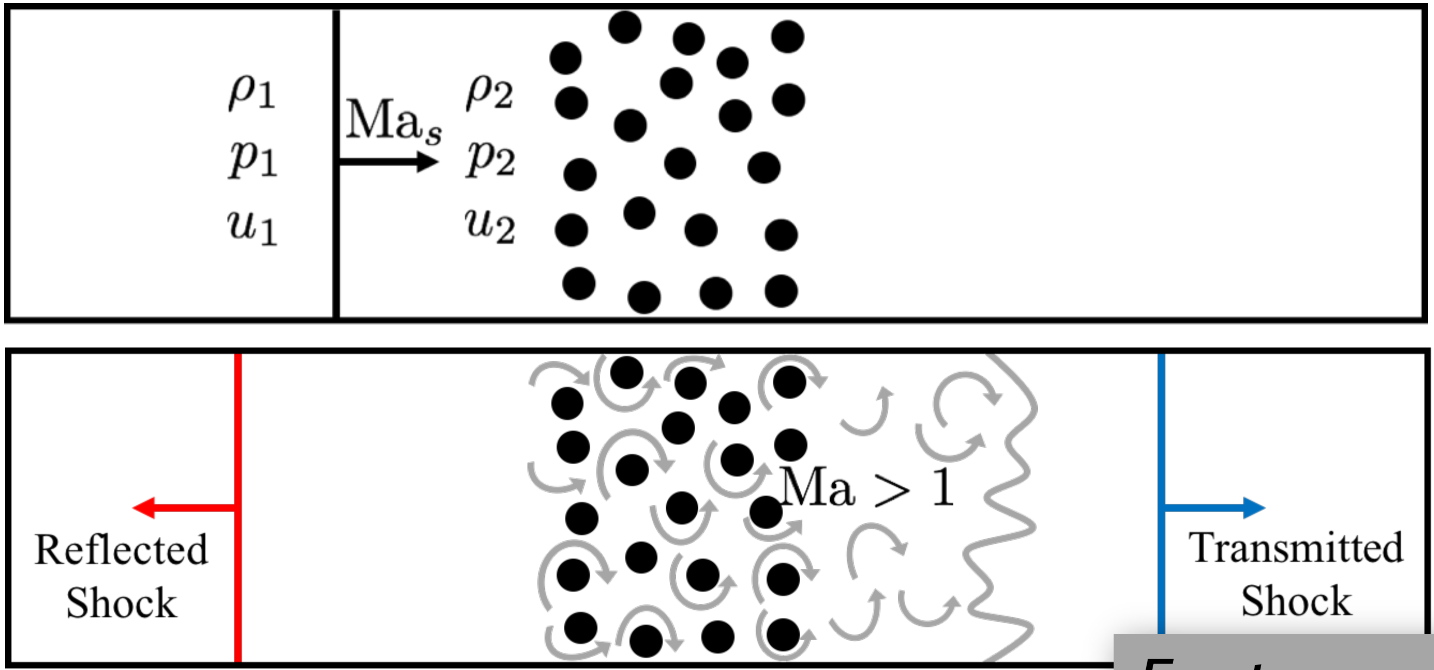
Sen et al. (2018), JFM.

Mehta et al. (2019), JFM.

Hosseinzadeh-Nik et al (2018), JFM.

[1]Shallcross, G. S., Fox, R. O., & Capecelatro, J. (2020). *Int. J. Multiphase Flow*, **122**, 103138

Schematic (shock-particle curtain)



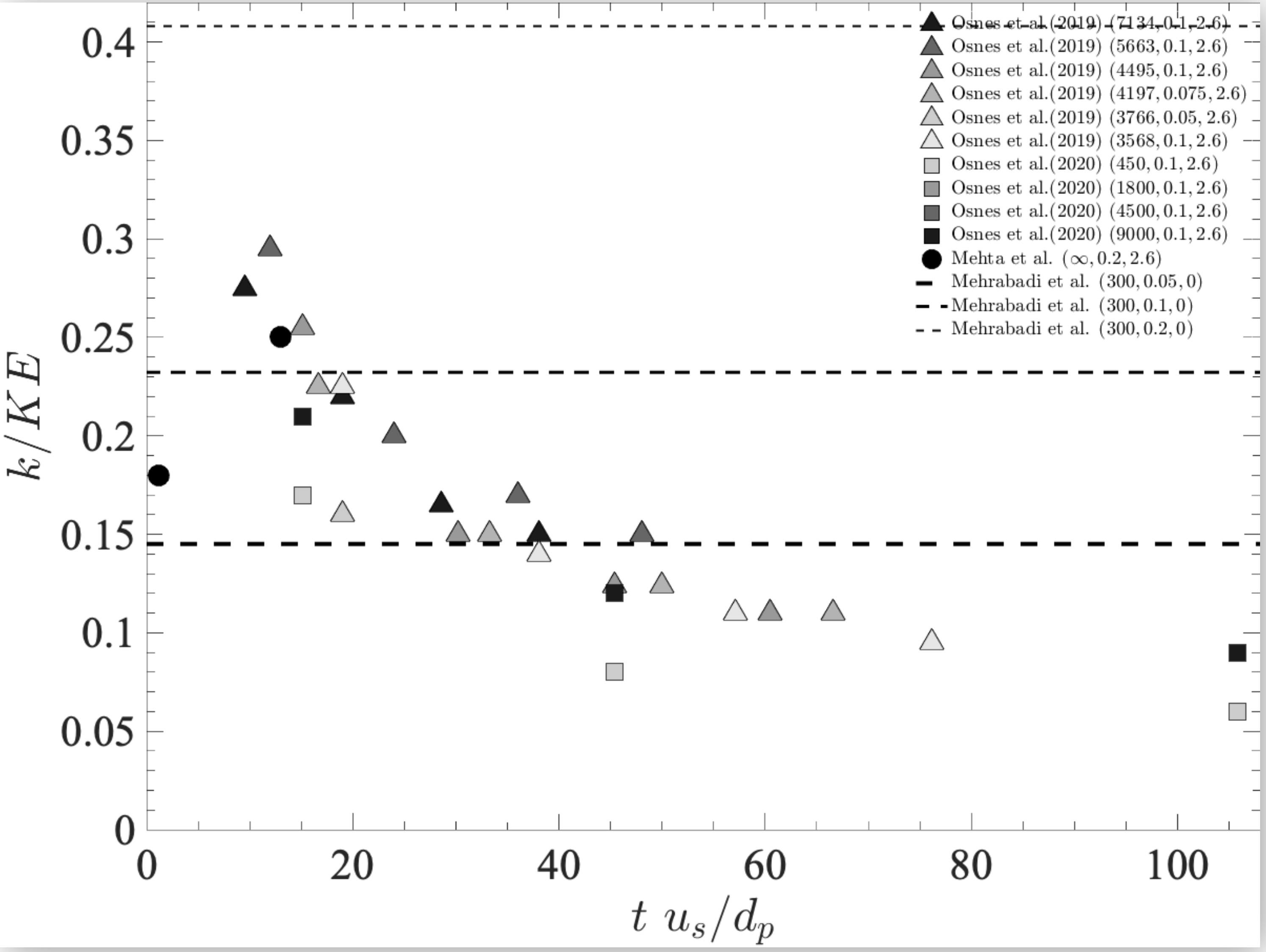
$t = 0$

$t = t_1$

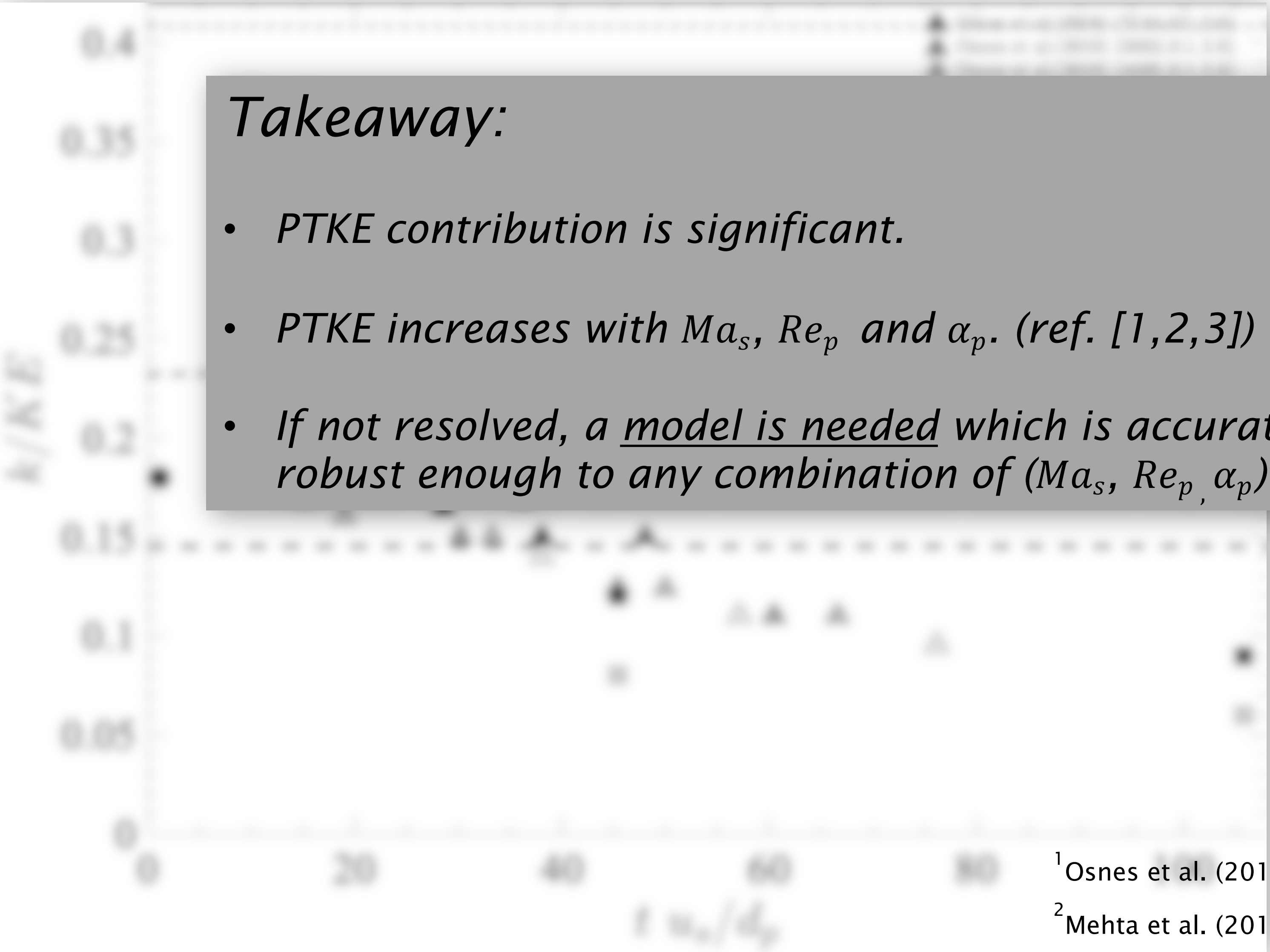
Features:

- Large slip velocities
- Wake-wave interaction
- Choking
- Locally supersonic zones
- Large gas phase acceleration

PTKE (Shock-particle cases)



PTKE (Shock-particle cases)



Takeaway:

- *PTKE contribution is significant.*
- *PTKE increases with Ma_s, Re_p and α_p. (ref. [1,2,3])*
- *If not resolved, a model is needed which is accurate and robust enough to any combination of (Ma_s, Re_p, α_p).*

[1] Osnes et al. (2019), IJMF.

[2] Mehta et al. (2019), PRF.

[3] Shallcross et al. (2020), IJMF.

Modeling efforts so far

- *For incompressible regime Mehrabadi et al.[1] proposed α_p, Rep dependent PTKE correlation.*
  - *Valid for $\alpha_p \leq 0.5$ and Re_p<300, will not predict PTKE downstream of particle curtain when $\alpha_p = 0$.*

- *Algebraic closure of PTKE by Osnes et al.[2]*
  - *Only accounts for PTKE contribution due to wake (PTKE contribution due to shock reflection from individual particle is neglected)*
  - *Validity is tested against 2< Ma_s<3, Re_p>3000, $\alpha_p \leq 0.1$ only.*

- *Shallcross et al.[3] proposed transport equation for PTKE.*
  - *Algebraic closure to dissipation of PTKE. Transporting PTKE is versatile, since it allows one to predict advected PTKE downstream as well in the absence of particles.*

Summary of limitations:

➢ *Works under limited conditions considered in the literature.*

- *There is a need for a robust closure model for PTKE!*

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[1] Mehrabadi et al. (2015), JFM.

[2] Osnes et al. (2019), PRF.

[3] Shallcross et al. (2020), IJMF.

Two Equation model

PTKE transport equation[1]

$$\frac{\partial \alpha\rho k}{\partial t} + \nabla\cdot(\alpha\rho\boldsymbol{u}k) = \underbrace{\mathcal{P}_S}_{-\alpha\boldsymbol{R}_u\,:\,\nabla\boldsymbol{u}} + \underbrace{\mathcal{P}_D}_{(\boldsymbol{u}_p-\boldsymbol{u})\cdot\boldsymbol{\mathcal{F}}} - \alpha\rho\overset{\text{dissipation rate}}{\varepsilon},$$

To address shortcomings of ad-hoc algebraic closure of ε by Shallcross et al.[1] *, we propose*

A transport equation for dissipation of PTKE

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[1] Shallcross et al. (2020), IJMF.

Two Equation model

PTKE transport equation[1]

$$\frac{\partial \alpha\rho k}{\partial t} + \nabla\cdot(\alpha\rho\boldsymbol{u}k) = \underset{\underset{-\alpha\boldsymbol{R}_u:\nabla\boldsymbol{u}}{\swarrow}}{\mathcal{P}_S} + \underset{\underset{(\boldsymbol{u}_p-\boldsymbol{u})\cdot\boldsymbol{\mathcal{F}}}{\searrow}}{\mathcal{P}_D} - \alpha\rho\overset{\overset{\text{dissipation rate}}{\uparrow}}{\varepsilon},$$

Dissipation of PTKE transport equation

$$\frac{\partial \alpha\rho\varepsilon}{\partial t} + \nabla\cdot(\alpha\rho\boldsymbol{u}\varepsilon) = C_{\varepsilon,1}\frac{\varepsilon}{k}\mathcal{P}_S + \frac{\mathcal{P}_D}{\underset{\underset{\text{dissipation rate time scale}}{\downarrow}}{\boxed{\tau_k}}} - \alpha\rho C_{\varepsilon,2}\frac{\varepsilon^2}{k}.$$

Or

$$\frac{\partial \alpha\rho\omega}{\partial t} + \nabla\cdot(\alpha\rho\boldsymbol{u}\omega) = C_{\omega,1}\frac{\omega}{k}\mathcal{P}_S + \frac{\mathcal{P}_D}{k\tau_k} - \left(C_{\omega,2} + C_{\omega,3}\frac{\mathcal{P}_D}{\alpha\rho k\omega}\right)\alpha\rho\omega^2.$$

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[1] Shallcross et al. (2020), IJMF.

Two Equation model for homogeneous case

PTKE transport eq.

$$\frac{\partial \alpha\rho k}{\partial t} + \nabla\cdot(\cancel{\alpha}\rho\boldsymbol{u}k) = \cancel{\mathcal{P}}_S + \mathcal{P}_D - \alpha\rho\varepsilon,$$

As a starting point, focusing on unsteady but spatially homogeneous case

Dissipation transport eq.

$$\frac{\partial \alpha\rho\varepsilon}{\partial t} + \nabla\cdot(\alpha\cancel{\rho}\boldsymbol{u}\varepsilon) = C_{\varepsilon,1}\cancel{\frac{\varepsilon}{k}}\mathcal{P}_S + \frac{\mathcal{P}_D}{\boxed{\tau_k}} - \alpha\rho C_{\varepsilon,2}\frac{\varepsilon^2}{k}.$$

$\Downarrow$

unclosed

needs closure model

Two Equation EL framework

Using extracted correlation for PTKE given by Mehrabadi et al.[1] *at steady state,*

$$\frac{2k}{||\boldsymbol{u}_p - \boldsymbol{u}||^2} = 2\alpha_p + 2.5\alpha_p\left(1-\alpha_p\right)^3 \exp\left(-\alpha_p \mathrm{Re}_p^{1/2}\right)$$

Then, using it in EL framework, the dissipation time scale can be,

$$\tau_k = \frac{\rho\tau_p}{\rho_p F C_{\varepsilon,2}}\left[1 + 1.125\left(1-\alpha_p\right)^3 \exp\left(-\alpha_p \mathrm{Re}_p^{1/2}\right)\right]$$

Assumptions:

- *Homogeneous and steady state*
- *Valid for $\alpha_p < 0.5$ and $Re_p < 300$*
- *Valid in incompressible limit*

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[1] Mehrabadi et al. (2015), JFM.

Towards closure model with finite Ma correction

Using PR-DNS to quantify the budget of PTKE over range of α_p and Ma

Flow is characterized by:

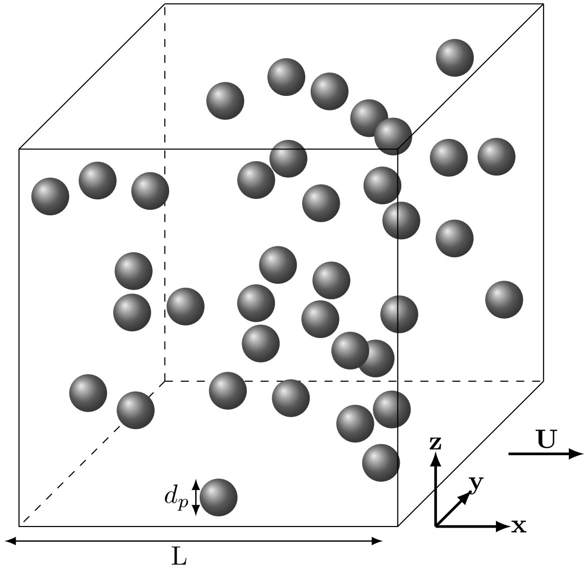
- Reynolds number based on superficial velocity:

$$Re_p = \frac{\rho(1-\alpha_p)|\boldsymbol{u}-\boldsymbol{u}_p|d_p}{\mu}$$

- Mach number:

$$\mathrm{Ma} = \frac{|\boldsymbol{u}-\boldsymbol{u}_p|}{c}$$

- Solid volume fraction: α_p



With:

- *Re_p fixed (Re_p=300)*
- *$0.1 \leq Ma \leq 1$*
- *$0.05 \leq \alpha_p \leq 0.4$*

Check out AIAA SciTech talk:

Quantification of drag statistics for compressible flows past particles at finite Mach number and volume fraction[2]

Session: FD-38 CFD methods IX

[2] Khalloufi et al. (2021), SciTech Forum.

Budget of PTKE

- *PR-DNS is used to inform the budget of PTKE to aid in model development.*

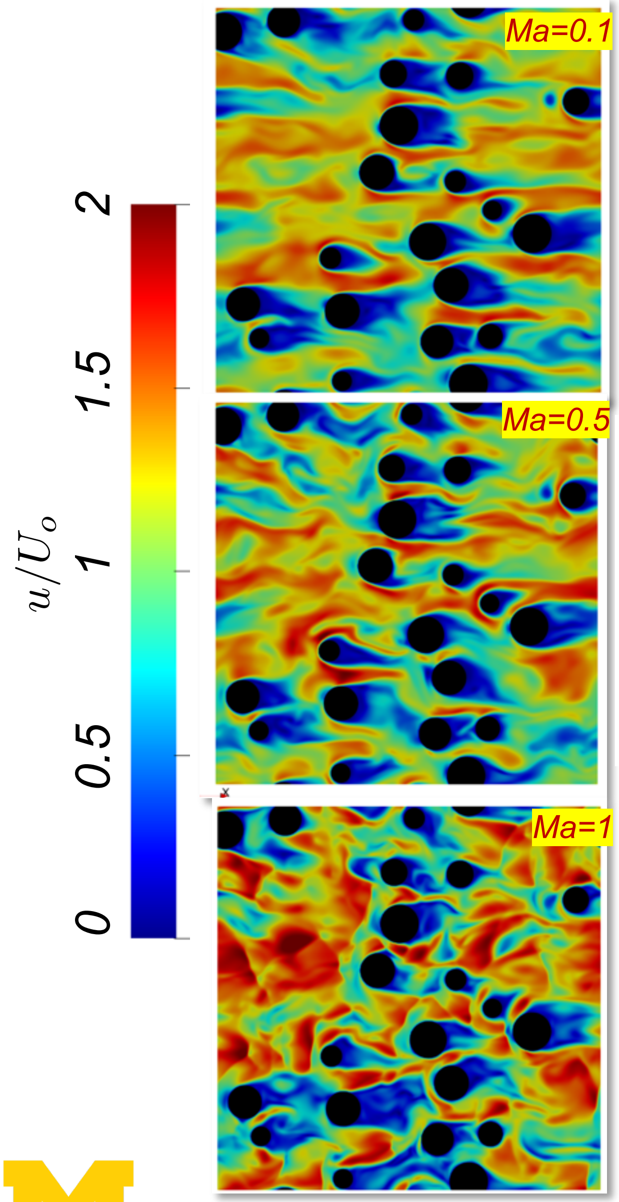
The evolution of PTKE in statistically homogeneous flows,

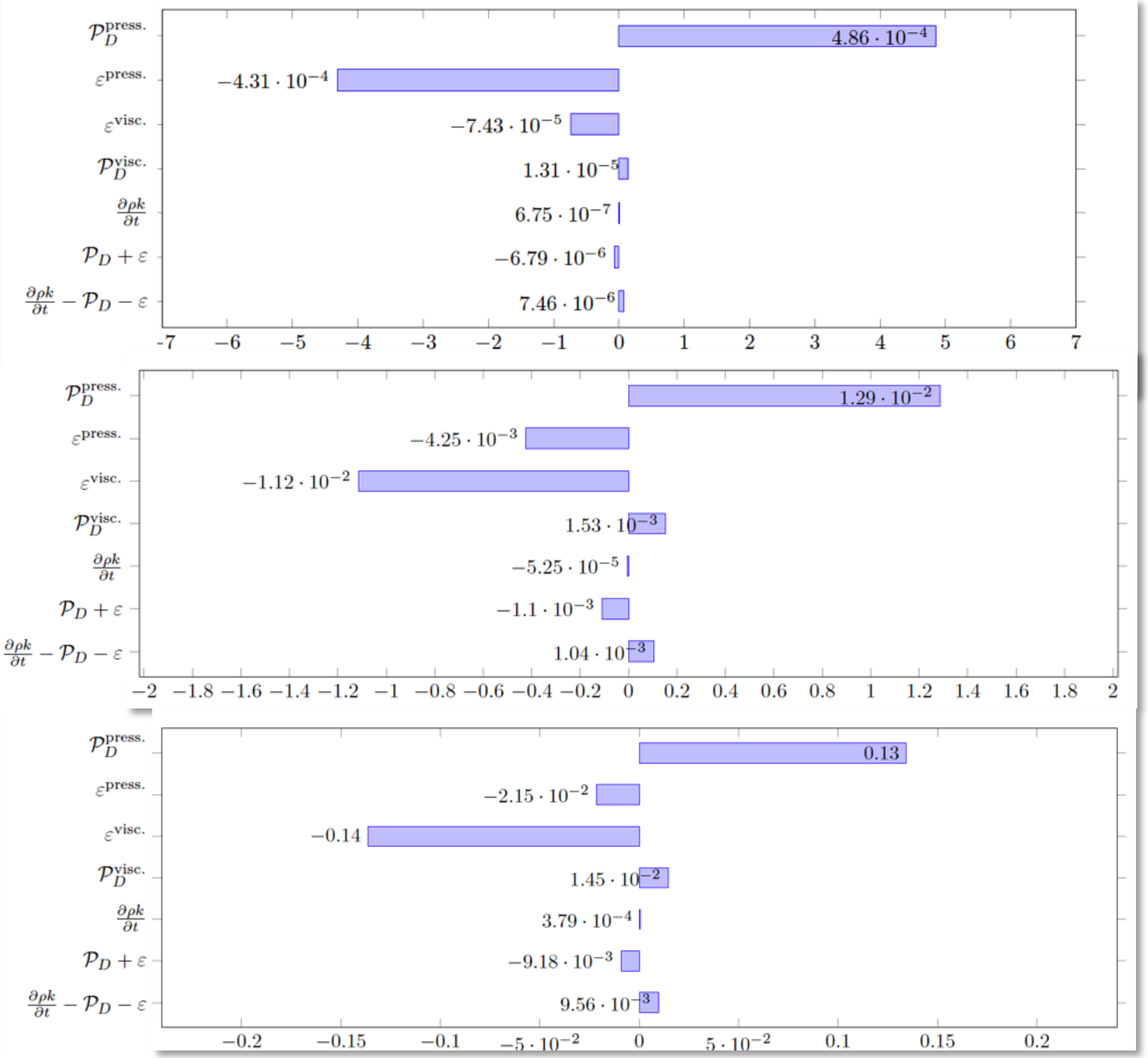
$$\langle\rho\rangle\alpha\frac{\mathrm{d}k}{\mathrm{d}t} = \underbrace{-\langle\boldsymbol{u}''\cdot(-p\mathrm{I}+\boldsymbol{\tau})\cdot\boldsymbol{n}\delta(\boldsymbol{x}-\boldsymbol{y})\rangle}_{\substack{\text{Production due to drag}\\(\mathcal{P}_D)}}\underbrace{-\langle\mathcal{I}(-p\mathrm{I}+\boldsymbol{\tau}):\nabla\boldsymbol{u}''\rangle}_{\substack{\text{Dissipation}\\(\varepsilon)}}$$

$$\mathcal{P}_D^{\text{press.}} = \langle\boldsymbol{u}''\cdot p\boldsymbol{n}\;\delta(\boldsymbol{x}-\boldsymbol{y})\rangle \qquad \varepsilon^{\text{press.}} = \langle\mathcal{I}p\nabla\cdot\boldsymbol{u}''\rangle$$

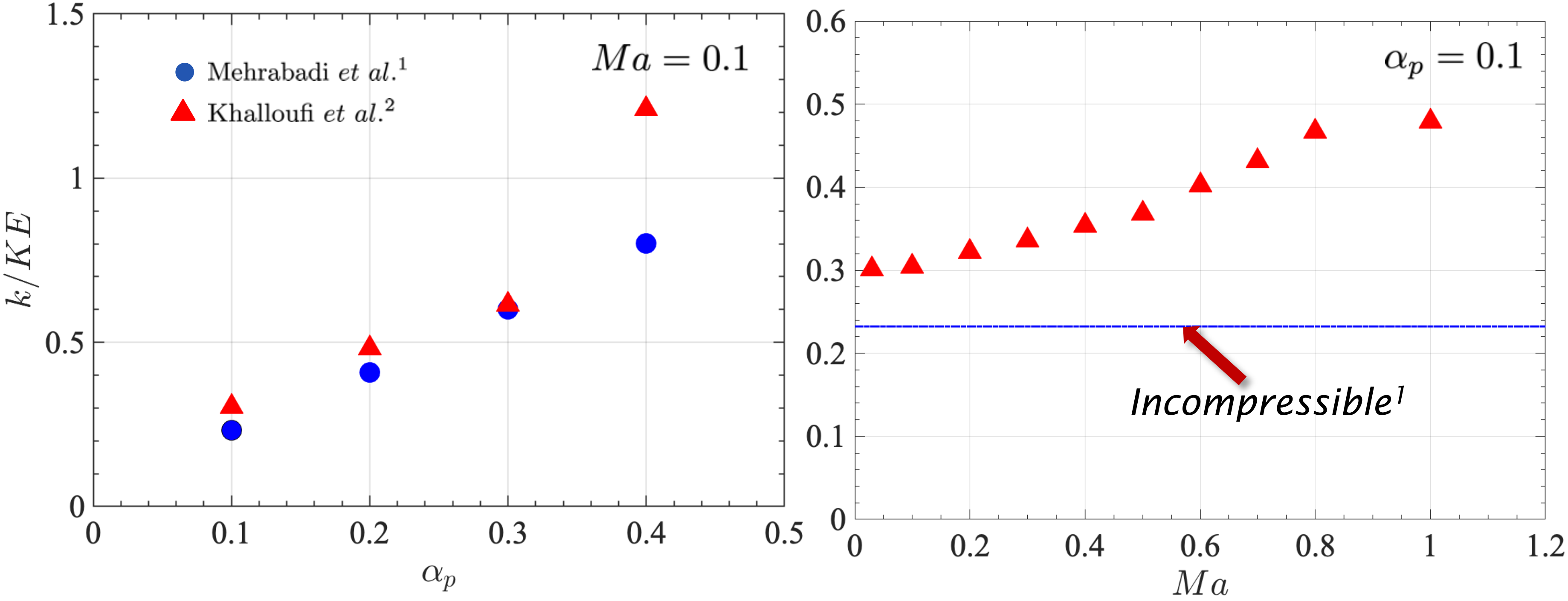
$$\mathcal{P}_D^{\text{visc.}} = -\langle\boldsymbol{u}''\cdot(\boldsymbol{\tau}\cdot\boldsymbol{n})\;\delta(\boldsymbol{x}-\boldsymbol{y})\rangle \qquad \varepsilon^{\text{visc.}} = -\langle\mathcal{I}\boldsymbol{\tau}:\nabla\boldsymbol{u}''\rangle$$

Quantified budget of PTKE ($\alpha_p = 0.1, Rep = 300$)





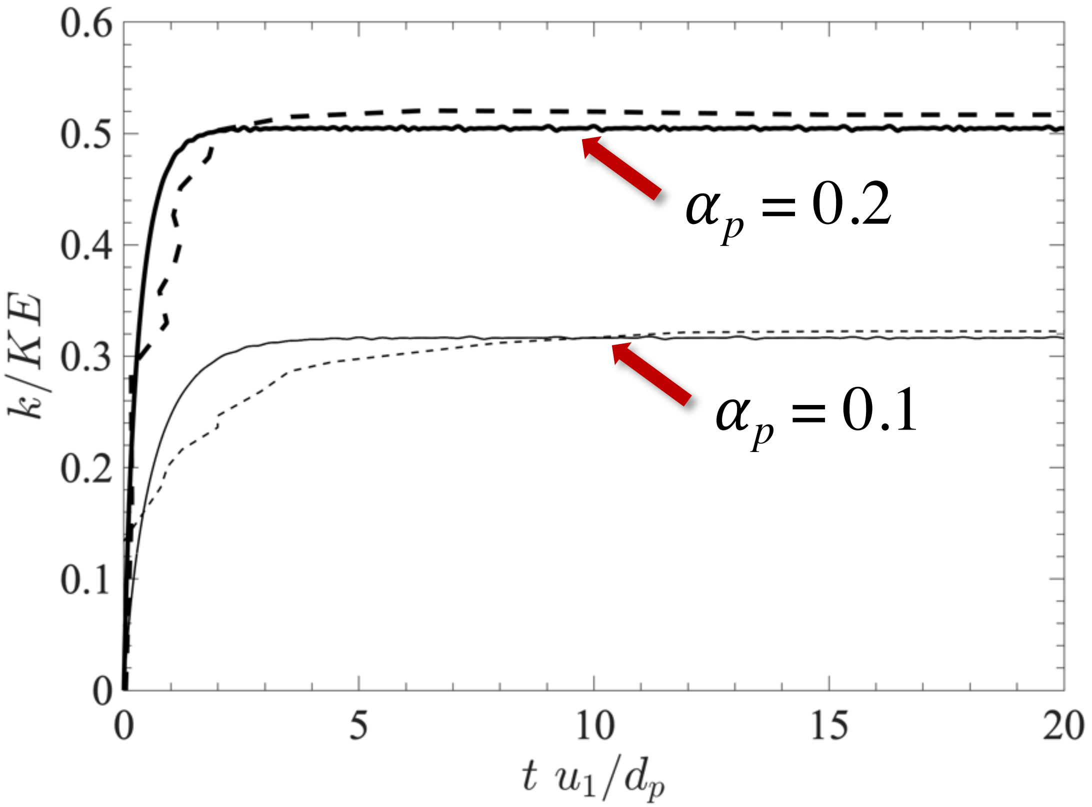
PTKE Vs Volume fraction, Ma



[1] Mehrabadi et al. (2015), JFM.

[2] Khalloufi et al. (2021), SciTech Forum.

Comparison with incompressible PR-DNS

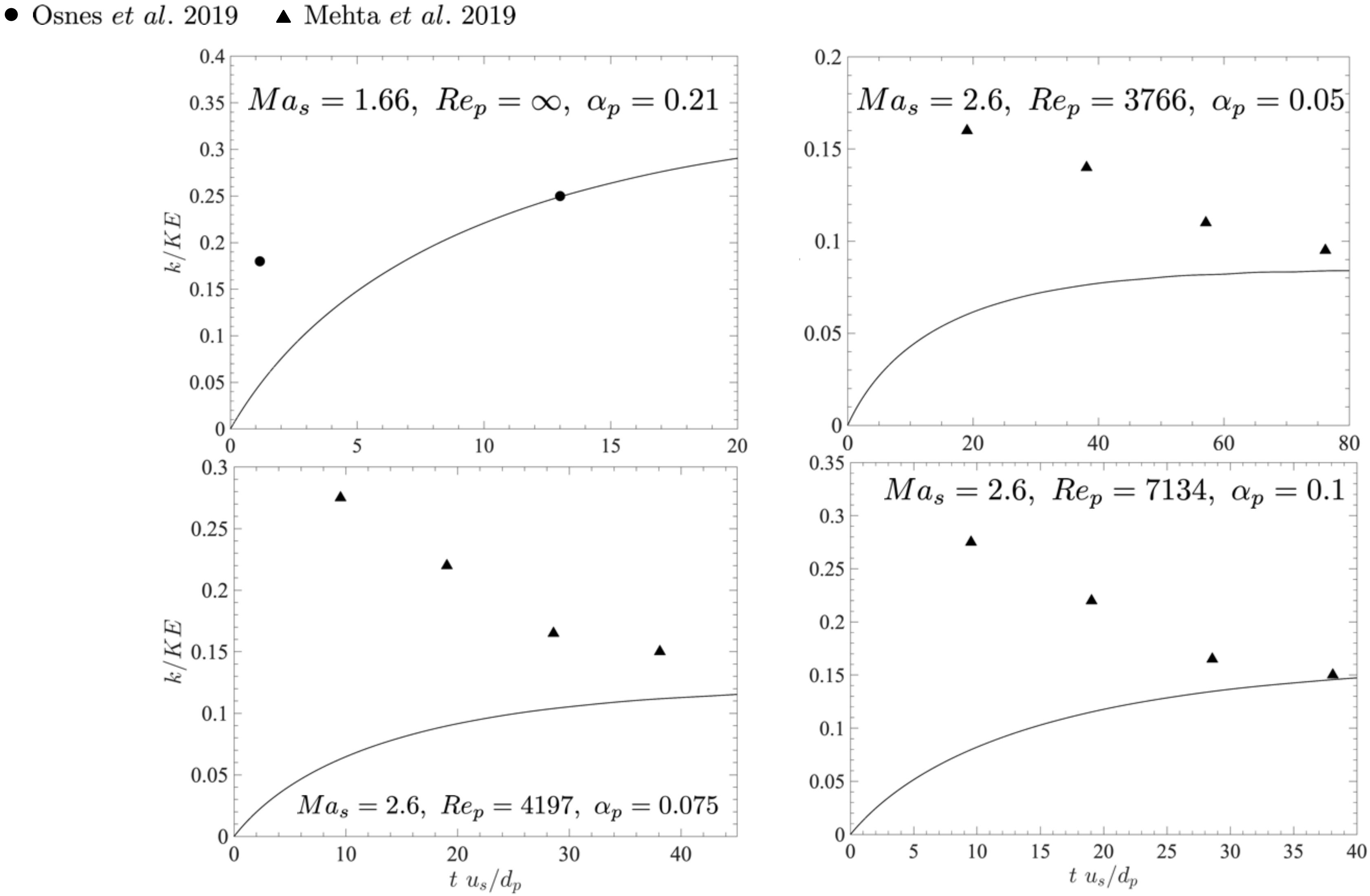


$Re_p = 20$

(——) *2 Eq EL*
(- - -) *Mehrabadi et al.*

Mehrabadi et al. (2015), JFM.

Comparison with shock-particle cases



Conclusion and Future work

- We seek to formulate an Eulerian-Lagrangian model for high-speed gas-particle flows. Pseudo turbulent Reynolds stress plays a key role and needs to be accounted for.
- We present a two-equation EL framework. Closure is needed for dissipation time scale.
- Particle-resolved simulations were performed under finite Mach number and volume fraction conditions for homogeneous gas-solid suspensions.
- The budget of PTKE shows drag production is balanced by viscous dissipation, pressure dilatation also plays a role.
- Preliminary closure informed by incompressible models fail to predict PTKE evolution from literature.
- Looking forward, use PR-DNS to inform closure for statistically stationary, homogeneous conditions.

Questions?