# Numerical Methods to Generate Solar Sail Trajectories

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# Why use numerical methods for solar sail trajectory design?

In search for new solar sail mission concepts, most trajectory work has been analytical

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**Numerical tools complement analytical techniques**

- May not require advanced knowledge of solution structure
- Expose new solutions
- Necessary for many mission applications
- Suite of tools required to meet different goals

# Exploring future trajectory options

*Address the questions: Where can sails go? What level of technology is required?*

Sailcraft trajectories are boundary value problems
and can be solved using numerical BVP-solving techniques

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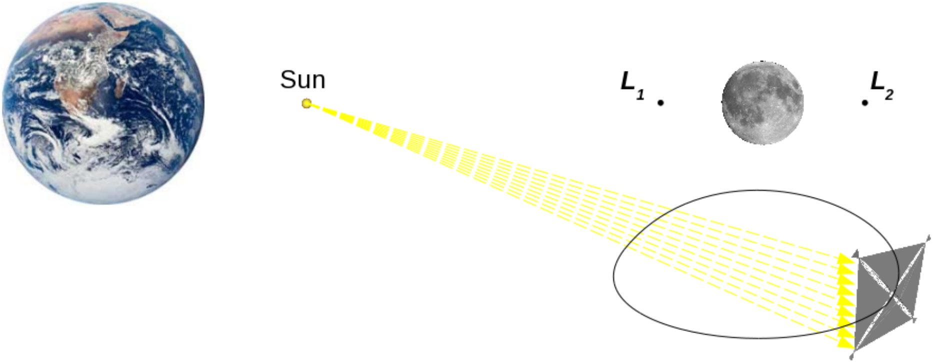
| **Shooting** | **Collocation** | **Finite-difference methods** |
| --- | --- | --- |
| Sun–Earth Halo orbits<br>Nuss (1998)<br>A. McInnes (2000) | Interplanetary trajectories<br>Melton (2002)<br>Nassiri et al. (2005) | Lunar south pole coverage<br>Wawrzyniak & Howell (2009) |
| Offset SE Halo orbits<br>Waters & C. McInnes (2007)<br>Farrés & Jorba (2010) | Lunar south pole coverage<br>Ozimek, Grebow & Howell (2008, 2009, 2009) | |
| | Levitated geostationary orbits<br>Baig & McInnes (2010) | |

# Can a sail solve the lunar south pole coverage problem?




Sailcraft in view of LSP (15° elev. constraint)
Earth and lunar gravity
No solar gravity, SRP only

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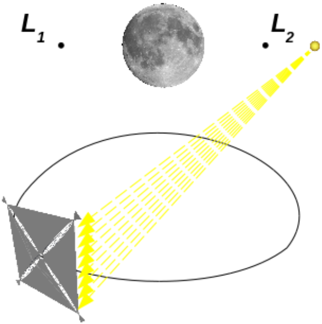
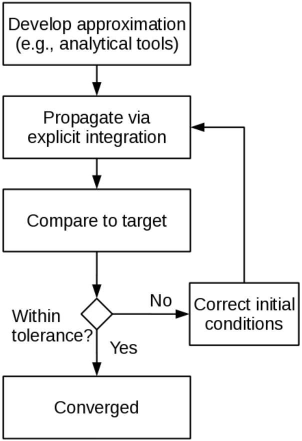


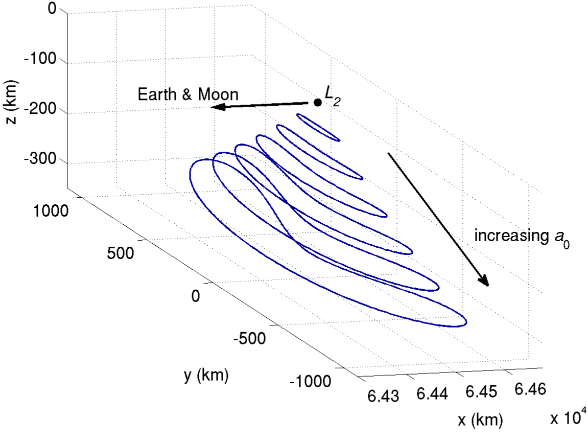

Sailcraft in view of LSP (15° elev. constraint)
Earth and lunar gravity
No solar gravity, SRP only
Sun moves with respect to fixed Earth and Moon

# Shooting methods (a.k.a. differential correctors)

*Develop analytical approximation, correct with (single) shooting*

*Fix attitude, only correcting path variables*




$a_0$: characteristic acceleration

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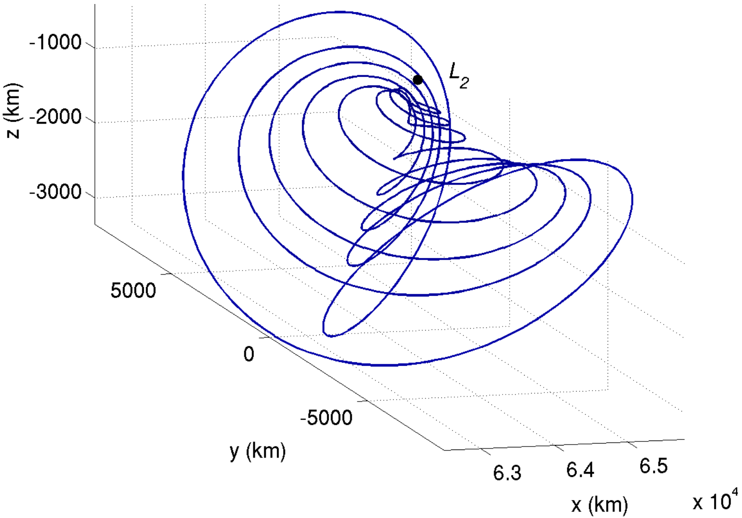
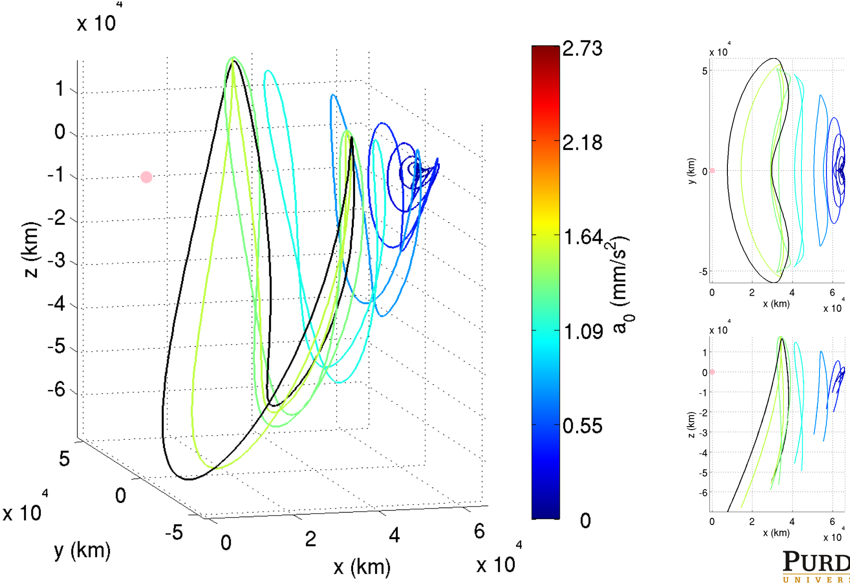
# Use continuation to predict subsequent orbits in family

*Characteristic acceleration ranges from 0.017 mm/s² to 0.118 mm/s²*



# Continue to first eigenvalue bifurcation

*Characteristic acceleration ranges from 0.12 mm/s² to 1.59 mm/s²*



# Finite-difference method

*Simple, simple, simple*

How it works:

- Guess a path and attitude profile
- Discretize guessed path
- Replace $\mathbf{a}_i$ and $\mathbf{v}_i$ in EOM at each epoch with central difference approximations based on guessed path
- Iterate until path and *variable* attitude profile satisfy EOM

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Why use it?

- Simple to understand and implement
  - Path constraints easily included
- Reasonable local accuracy: $\mathcal{O}(\Delta t^2)$
- Millions of solutions available quickly
- Survey the design space, unveil new solutions
  - To satisfy 15° elevation constraint, $a_0 > 1.5$ mm/s$^2$

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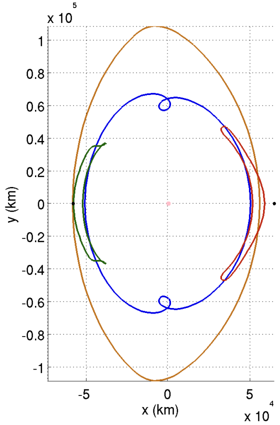
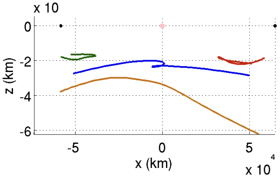
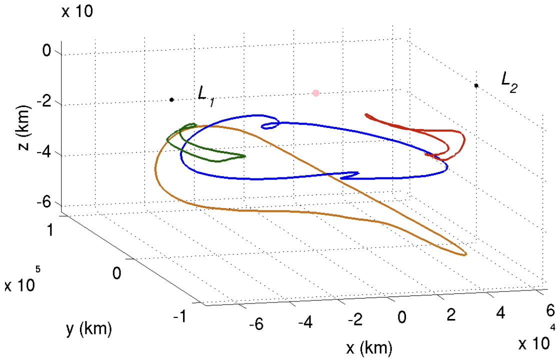
# Example FDM solutions

*Each meets 15° elevation constraint,* $a_0 = 1.7\ \mathrm{mm/s^2}$



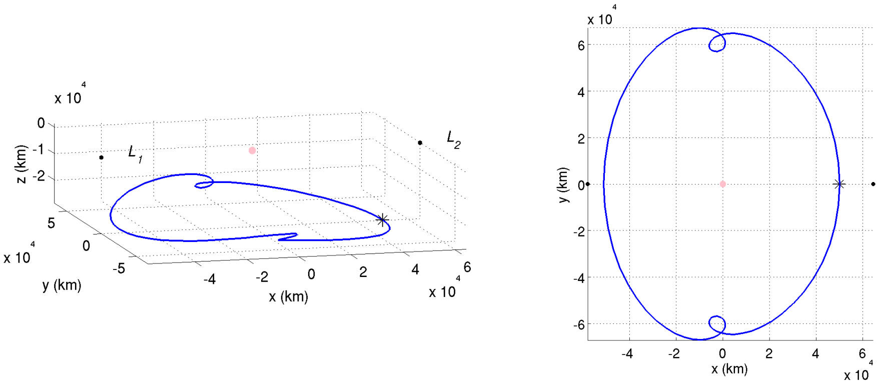




All orbits have periods of 29.5 days

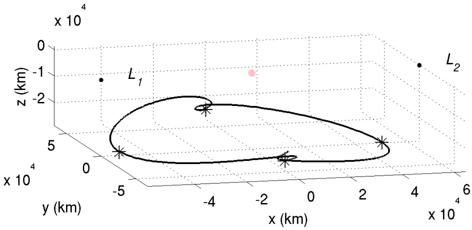
# Bootstrapping: use FDM solution to initialize shooter

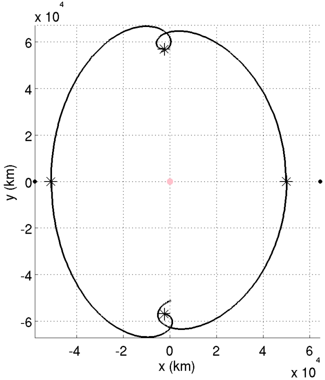
*Start with reference orbit from FDM*



# Bootstrapping: use FDM solution to initialize shooter

*Propagate from 4 states along reference trajectory*

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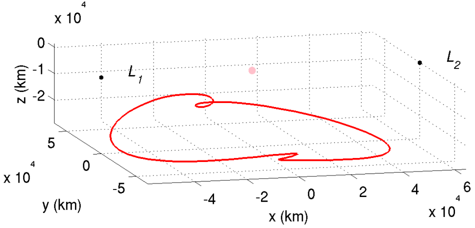
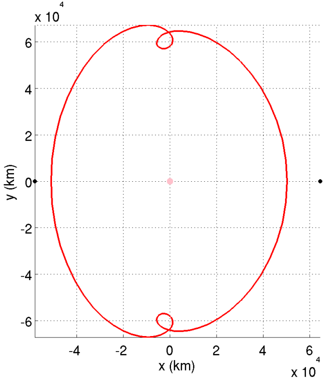
# Bootstrapping: use FDM solution to initialize shooter

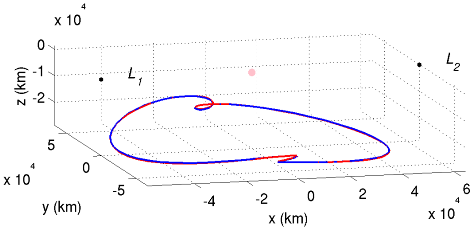
*Correct until interior nodes are continuous*

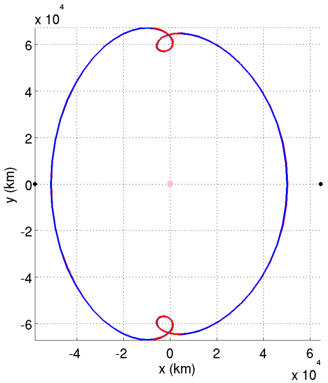


# Bootstrapping: use FDM solution to initialize shooter

*Solution resembles reference*



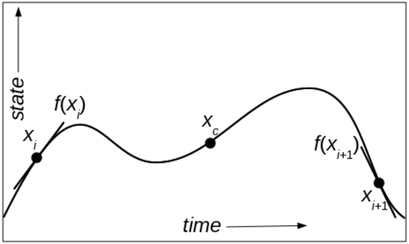



# Collocation

*Elegant, accurate, slightly more complicated*

How it works:

- Discretize a guessed path and attitude profile
- Fit $n^{\text{th}}$-degree polynomial in sub-arcs between nodes
  - May require internal points, depending on $n$




Fourth-degree polynomial...

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- Fit $n^{\text{th}}$-degree polynomial in sub-arcs between nodes
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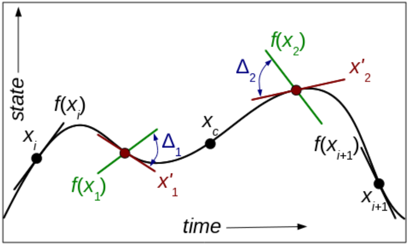
- Compare EOM to derivative of polynomial at defect point(s) between nodes
- Iterate until defects ($\Delta_{1,2}$) are zero




...with defect (collocation) points

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# Collocation

*Elegant, accurate, slightly more complicated*

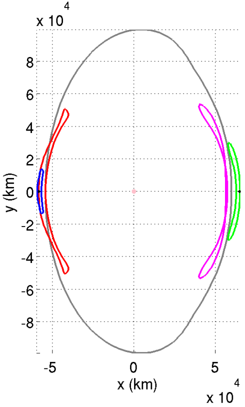
Why use it?

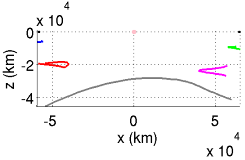
- Can include path constraints
- Results in trajectory *and* attitude profile
- Accuracy improves as polynomial degree increases

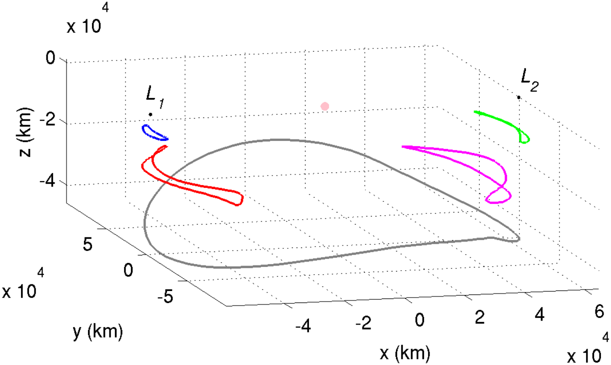
  $n = 0$, $\mathcal{O}(\Delta t^1)$. $n = 2$, $\mathcal{O}(\Delta t^3)$.
  $n = 3$, $\mathcal{O}(\Delta t^5)$. $n = 4$, $\mathcal{O}(\Delta t^7)$.
  $n = 5$, $\mathcal{O}(\Delta t^9)$. $n = 7$, $\mathcal{O}(\Delta t^{13})$.

# Propagate states from collocation solutions

*Ozimek et al. (2009) provide states and control laws for explicit integration from solution using a 7$^{\text{th}}$-degree polynomial and Gauss-Lobatto integration constraints*

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# Summary of numerical tools

*Different techniques useful at every level of mission design*

Single shooting

- Uses knowledge of solution shape and dynamical properties

Multiple shooting

- Improved numerical stability

Finite-difference method

- Simple; allows crude initial guess

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Numerical methods

- Accuracy of solutions only as good as fidelity of model
- Yield input to higher-fidelity models
- Great starting point for understanding design space

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# Conclusion

Add numerical methods to the
solar sail trajectory design toolbox

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